Final Exam Review - Day 2 - 12/6/2023

Final Exam

Posted Dec 4, 2023 10:29 AM

The Final Exam will be on Friday, December 15 from 10:30am-12:30pm. We will all be in Loeb Playhouse (STEW 183). When you enter the lobby of Loeb, you will need to pick up a lapboard for taking the exam. You will be on either the main floor or the balcony, depending on your instructor.

Main Floor: Victor Hughes, Jakayla Robbins, Alexandra Cuadra

Balcony: Ben Doyle, Dave Norris

There are no old published finals available, but there is plenty of practice in LON-CAPA. The material on the final exam is evenly distributed among the topics covered this semester. There is an Exam Memo posted in Contents. On the second page of the exam memo is the formula sheet that you will be given on the exam (it will be attached at the back of the exam).

Attachment(s):

- FinalExamMemo.pdf (49.88 KB)
- 16020_final-f23_merged.pdf (286.66 KB)

Formula Sheet - MA 16020 Final Exam

Geometric Series:

The geometric series $\sum_{n=0}^{\infty} ar^n$ with common ratio r converges if $|r| < 1$ with the sum

$$S = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

Power Series/Maclaurin Series:

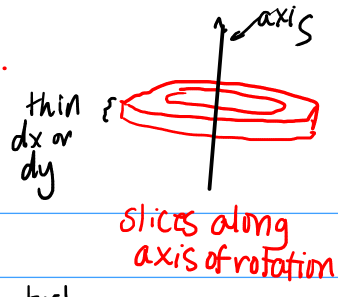
$$\begin{aligned}
\frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + ..., |x < 1| \\
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ... \\
\sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... \\
\cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... \\
\ln(1+x) &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...
\end{aligned}$$

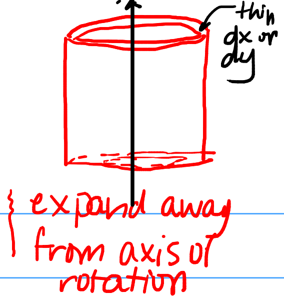
Second Derivative Test

Given the critical point (a, b), such that $f_x(a, b) = 0$ and $f_y(a, b) = 0$, and let

$$D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

- If $D > 0$ and $f_{xx}(a, b) > 0$ then $f(a, b)$ is a relative minimum.
- If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a relative maximum.
- If $D < 0$, then $f(a, b)$ is a saddle point.





Volume Review.

Disks: $V = \int_a^b \pi r^2 \, dx/dy$

Washers: $V = \int_a^b \pi (R^2 - r^2) \, dx/dy$

Shells: $V = \int_a^b 2\pi \, r \, h \quad dx/dy$

r ↑ easy, h ↑ harder

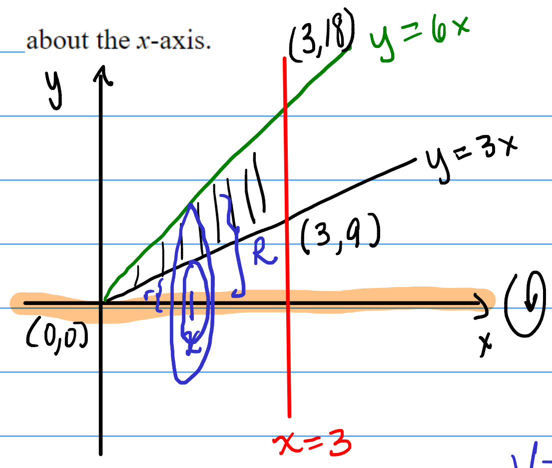
axis \ tool	Disks/ Washers	Shells
vertical (like y-axis) $x=0$	dy	dx
horizontal (like x-axis) $y=0$	dx	dy

Exam Review, Lesson 15, #4

Find the volume of the solid obtained by revolving the region enclosed by the curves

$$y = 3x, y = 6x \text{ and } x = 3$$

about the x-axis.



washers

horizontal axis (x-axis) → dx

$R = 6x$ $\qquad$ $x: 0 \to 3$

$r = 3x$

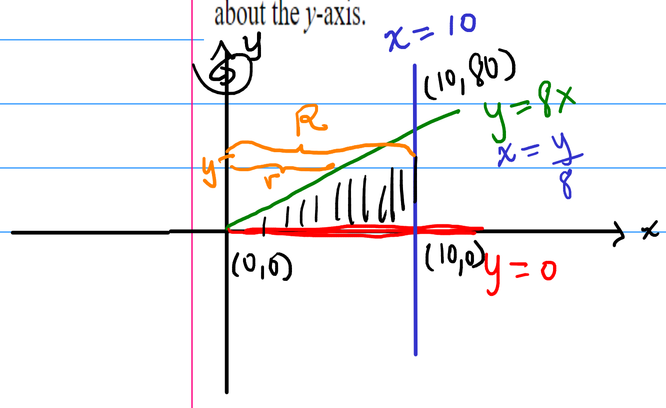
$$V = \int_0^3 \pi \left((6x)^2 - (3x)^2 \right) dx$$

Ex Review, Lesson 17 #1

Find the volume of the solid obtained by revolving the region enclosed by the curves

$$y = 8x, y = 0 \text{ and } x = 10$$

about the y-axis.

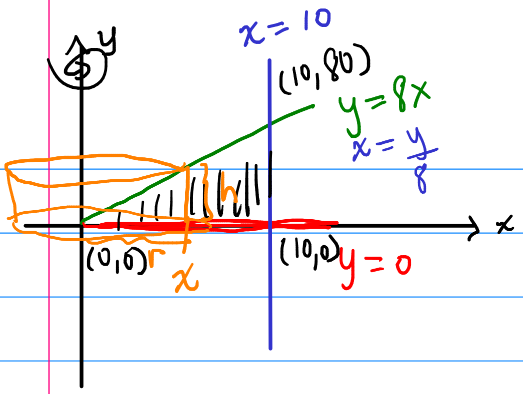


washers: vert. axis → dy

$y: 0 \to 80$

$R = 10 \qquad r = \frac{y}{8}$

$$V = \int_0^{80} \pi \left(10^2 - \left(\frac{y}{8}\right)^2 \right) dy$$



Shells: vert. axis $\longrightarrow$ dx

$x: 0 \to 10$

$r = x$

$h = (8x - 0) = 8x$

green red

$$V = \int_0^{10} 2\pi x\,(8x)\,dx = \int_0^{10} 16\pi x^2\,dx$$

Partial Fraction Decomposition Review

Rational fctn. $\frac{P(x)}{Q(x)}$ $\longrightarrow$ factor $Q(x)$ as much as possible

factors will be linear: $ax+b$

and irreducible quadratic: ax^2+bx+c

$b^2-4ac<0$

$(ax+b)^n$ factor $\longrightarrow$ $\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_n}{(ax+b)^n}$

$(ax^2+bx+c)^n$ factor $\longrightarrow$ $\frac{B_1x+C_1}{ax^2+bx+c} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \cdots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}$

w/ $b^2-4ac<0$

EX x^2+4, x^2+1, x^2+5

Test Review, Lesson 9 #1

Which of the following could represent a partial fraction decomposition of the given function?

$$f(x) = \frac{39}{7x^3 - 3x^2} = \frac{39}{x^2(7x-3)}$$

Please note that you only have two answer attempts.

- ○ $\frac{Ax+B}{x^2} + \frac{C}{7x-3}$
- ○ $\frac{A}{x} + \frac{B}{x} + \frac{C}{7x-3}$
- ○ $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{7x-3}$
- ○ $\frac{A}{x} + \frac{Bx+C}{x^2} + \frac{D}{7x-3}$
- ○ $\frac{A}{x} + \frac{Bx+C}{x^2} + \frac{Dx+E}{7x-3}$

$(7x-3)$ – linear $\rightarrow \frac{C}{7x-3}$

$x^2 = (x)^2 = (1x+0)^2 \rightarrow \frac{A}{x} + \frac{B}{x^2}$

Common mistake is to think that x^2 is ~~irreducible quadratic~~

In fact, x^2 is linear squared.

PFD $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{7x-3}$

Ex. Review, Lesson 9, #3.

Find the partial fraction decomposition of the following expression. (Do NOT evaluate as an integral).

$$\frac{13x+112}{x^2+17x+70} = \frac{13x+112}{(x+7)(x+10)}$$

$$\frac{13x+112}{(x+7)(x+10)} = \frac{A}{x+7} + \frac{B}{x+10} = \frac{A(x+10)+B(x+7)}{(x+7)(x+10)}$$

linear

$13x+112 = A(x+10) + B(x+7)$ equal for all x values

2 easy x-values

$x = -10$

$x = -7$

$x = -10$

$13(-10)+112 = A(-10+10) + B(-10+7)$

$-130+112 = B(-3)$

$-18 = -3B$

$\boxed{B = 6}$

$x = -7$

$13(-7)+112 = A(-7+10) + B(-7+7)$

$21 = 3A$

$\boxed{A = 7}$

$$\int \left(\frac{7}{x+7} + \frac{6}{x+10} \right) dx$$

$$= 7\ln|x+7| + 6\ln|x+10| + C$$